dency toward seedlessness is seen in many cultivated plants, of which the potato is a good example. Since new varieties come mostly from seeds, many persons have supposed that plant-breeding must eventually cease in these plants; but the speaker pointed out that the constant choice of seeds for sowing is itself a powerful agent in conserving the seed-producing power of the plant. So long as we select seeds, so long we may expect the effects of this selection to give seeds in at least a part of the individuals of every generation.

Distribution of plants on fresh-water islands: By CONWAY MACMILLAN.

The islands in Lake of the Woods were selected for study and description, and, after a general account of the geology and physiography of this body of water and its surroundings, a classification of islands was proposed as follows: 1. Floating bog islands; 2. Scirpus-bar islands; 3. Sand-dune islands; 4. Irregular rock islands; 5. Dome-shaped rock islands.

The paper proposed a classification of strand plants and surf plants, and laid particular stress upon an interesting zonal distribution of plants which characterized the dome-shaped islands. In these it was shown that an outer ring of shrub was succeeded towards the center by a zone of trees, an inner zone of shrub and a central meadow or shrub. This distribution was explained as resulting from the silting off of soil, until a thicker ring of soil was formed at the periphery of the islands.

The paper was illustrated by fifty lantern slides, showing island landscapes and calculated to bring out the points made concerning strand plants and zonal distribution, together with views of islands in which irregularity of surface prevented zonal and promoted crevice distribution.

GEO. F. ATKINSON,
Secretary.
CORNELL UNIVERSITY

THE THIRD SUMMER MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

THE Third Summer Meeting of the American Mathematical Society was held in the lecture hall of the Buffalo Society of Natural Sciences, at Buffalo, N. Y., on August 31st and September 1st. Among those present were:

Dr. E. M. Blake, Prof. M. Bôcher, Mr. J. M. Brooks, Prof. F. N. Cole, Prof. J. E. Davies, Prof. A. T. DeLury, Prof. E. W. Davis, Dr. L. E. Dickson, Prof. W. P. Durfee, Prof. H. T. Eddy, Prof. T. S. Fiske, Miss Ida Griffiths, Dr. G. W. Hill, Dr. J. E. Hill, Dr. J. I. Hutchinson, Prof. E. W. Hyde, Prof. T. F. Holgote, Mr. P. A. Lambert, Dr. G. H. Ling, Prof. J. McMahon, Prof. M. Merriman, Prof. E. H. Moore, Prof. W. F. Osgood, Prof J. P. Pierpont, Dr. V. Snyder, Mr. W. M. Strong, Prof. O. Schmiedel, Prof. L. G. Weld, Prof. H. S. White, Prof. C. B. Williams, Miss E. C. Williams, Miss M. F. Winston, Prof. F. S. Woods and Prof. A. Ziwet.

The President, Dr. G. W. Hill, occupied the chair. Two sessions were held each day, beginning respectively at 10 a. m., and 2:30 p. m. The following papers were read:

1. *Methods of defining monogenic functions.* DR. E. M. BLAKE.
2. *An existence theorem for a class of linear euthymorphic functions of a single variable.* DR. E. M. BLAKE.
3. *A geometric method for the study of uniform convergence and certain double limits.* PROF. W. F. OSGOOD.
4. *Non-uniform convergence and the integration of series term by term.* PROF. W. F. OSGOOD.
5. *Two triply-infinite systems of simple groups.* DR. L. E. DICKSON.
6. *Ternray algebras.* PROF. J. B. SHAW.
7. *The geometry upon three surfaces of the seventh order.* DR. J. E. HILL.
8. *A special form of a quartic surface.* DR. J. I. HUTCHINSON.
9. *Note on the integral and integro-geometric series.* PROF. E. D. ROE.
10. *The cross ratio group of n! (n-3)-ic Cremona transportations of flat space of n-3 dimensions.* PROF. E. H. MOORE.
11. *Criteria for the reality of nodes in Dupin's cyclides, with a corresponding classification.* DR. V. SNYDER.
12. *Numerically regular reticulations upon surfaces of deficiency higher than 1.* PROF. H. S. WHITE.
13. *Loci of the equations* $p=\phi^x e$ *and* $p=\phi^{\alpha x}\psi^y e$ PROF. E. W. HYDE.

14. *On the hypothesis of the successive transmission of gravity and the possible perturbative effect on the earth's orbit.* PROF. J. MCMAHON.
15. *The continuity of chance.* PROF. E. W. DAVIS.
16. *A method of finding without a table the number corresponding to a given logarithm.* DR. ARTEMAS MARTIN.
17. *Table giving the first forty roots of the Bessel equation* $J_0(x)=0$, *and the corresponding values of* $J_1(x)$ PROF. B. O. PEIRCE and MR. R. W. WILSON.
18. *On the projective group.* PROF. H. TABER.

Dr. Blake's first paper gave a classification of the methods which have been used for defining monogenic functions, with example, references and historical notes. His second paper contained a demonstration of the existence of monogenic functions satisfying a relation which had presented itself as a greatly generalized form of the addition theorem of elliptic functions.

Prof. Osgood's first paper gave a geometric method, consisting in the study of the approximation of curves $y=S_n(x)$, for discussing the manner in which $S_n(x)$ converges (uniformly or non-uniformly) toward its limit $f(x)$ when $n=\infty$. Applications were made to the allied problems in double-limits of integrating and differentiating a series term by term. Numerous examples served as illustrations. His second paper studied the most general manner of the convergence of $S_n(x)$ toward its limit $f(x)$, where $S_n(x), f(x)$ are continuous functions, and treated the problem of determining when

$$\int_{x_0}^{x} f(x)\,dx = \lim_{n=\infty} \int_{x_0}^{x} S_n(x)\,dx.$$

Broader sufficient conditions than those generally known for the integration of a series of continuous functions term by term were obtained. The former of these papers will appear in the *Bulletin of the American Mathematical Society* and the latter in the *American Journal of Mathematics*.

Jordon's decomposition of the general linear group on m indices, and of the group of Abelian substitutions on $2m$ indices, leads to two doubly-infinite systems of simple groups. By generalizing to the Galois field of order p^n, p being prime, Dr. Dickson reaches two systems of simple groups whose orders depend on three independent parameters m, n and p. His paper is intended for the *Annals of Mathematics*.

Prof. Shaw considered, in his paper, all the algebras in which three independent units occur. Dr. Hill used the general cubo-cubic transformation in three cases where the principal systems of the two spaces degenerate, to pass from a general cubic surface to a septic surface. In the first case the surface is distinguished by three triple and three double lines; in the second by a triple line and a double quintic, and in the third by a triple conic and a double quartic of the second kind. The quartic surface discussed in Dr. Hutchinson's paper is the locus of the vertex of a cone of the second order passing through six given points. If these six points be in involution on the twisted cubic through them, the coordinates of a point of the surface can be expressed in terms of elliptic functions. The object of Prof. Roe's paper was to obtain general formulæ for the summation of the integral and integro-geometric series.

All the cross-ratios of n quantities are expressible rationally in terms of any $n-3$ independent ratios. Starting with a certain such system of $n-3$ rations, and permuting the quantities one obtains in all $n!$ systems of the same type. The expression of these $n!$ systems in terms of any one leads to the group studied in Prof. Moore's paper. For $n=4$ it is the well-known group of six linear practical substitutions generated by $\lambda'=1/\lambda$, $\lambda'=1-\lambda$. For $n\geq 5$ it is holoedrically isomorphic to the symmetric group in n letters. It contains a sub-group of $(n-1)!$ collineations permuting amongst themselves certain $n-1$ fundamental points, and first given by Klein. Prof. Moore's group results from extending that of Prof. Klein by an

'inversion' having $n-2$ of the fundamental points as critical points and the remaining one as a fixed point. This paper is intended for the *Mathematische Annalen*.

In Dr. Snyder's paper Lie's haxaspherical coordinates were employed to define, without the use of line-geometry, a Dupin's cyclide. By associating the three simultaneous linear equations of definition with the point-complex and the plane-complex, one obtains determinants the signs of which indicate the reality of spheres common to the four complexes and thus show the presence of nodes. Dr. Snyder's paper will appear in the *Annals of Mathematics*.

On a convex surface of deficiency zero Euler's equation, together with the requirement of numerical regularity, gives three sets of integers for vertices, faces and edges of a polyedron: These by duality become five, corresponding to the five regular polyedra. On a surface of deficiency greater than unity the modified equation of Euler, together with similar limitations, gives again a finite number of sets of integers for vertices, faces and edges. These sets are of two sorts: 'derivative,' obtained from sets belonging to lower deficiencies; and 'special,' not so obtainable, but peculiar to the deficiency in question. These sets of characteristic numbers can be realized on concrete models. Prof. White discussed the subject in detail and exhibited models, constructed by Mr. O. H. Basquin, for deficiency 2 and for the 'special' sets of deficiency 3. For deficiency 2 there were 13 card models and 6 of plaster; for deficiency 3 there were 7 card models. Prof. White's paper will appear in the *Bulletin of the American Mathematical Society*.

In Prof. Hyde's paper, which is intended for the *Annals of Mathematics*, p is a variable point, e is a fixed point, x and y are scalars varying from $-\infty$ to $+\infty$, and φ and ψ are linear points, functions of the form:

$$\phi q = \Sigma (A_k e_k . e_k \mid q) \ , \ \psi q = \Sigma (B_k e_k . e_k \mid q).$$

The curve and surface represented by the equations given in the title of the paper are studied, and many interesting properties which they possess are described.

In Prof. McMahon's paper, also intended for the *Annals of Mathematics*, the sun is assumed to move in a straight line with constant velocity, which is shared by the whole solar system; and the gravitational influence is supposed to issue from the sun in waves that move outward with constant velocity (equals, perhaps, to that of light). When any wave reaches the earth the latter is attracted towards the wave center, or point of space from which the wave issued. This effective center of acceleration is at a distance from the sun which varies between the limits $ka\ (1-e)$ and $ka\ (1+e)$, where k is the ratio of the velocity of the sun to that of gravitation, a is the semi-axis major of the earth's orbit, and e the eccentricity. Thus the orbit of the earth relatively to the sun is that which would be due to a center of force that performs small oscillations about its mean position. The law of this oscillatory motion is first determined, and then the equations of acceleration of the earth in its orbit, along and perpendicular to the radius-vector, are corrected for this small disturbance; and appropriate solutions of these differential equations are given, correct as far as terms in ke^2. The most important perturbative terms are examined, and their effect on the orbit determined.

Prof. Davis maintained that there existed a necessary and intimate association between the notions of continuity and chance. His paper will be published in the *Nebraska University Studies*. Dr. Martin's paper contained several series suitable for calculating a number when its logarithm is given. He intends to publish the paper in the *Mathematical Magazine*.

The first ten values of x, for which Bessel's function of the zeroth order $J_0(x)$ vanishes, were given to ten places of decimals

by Meissel. The next thirty roots of the equation $J_0(x)=o$, and the values of $J_1(x)$ corresponding to the first forty roots, have just been computed by Prof. B. O. Peirce and Mr. R. W. Willson by means of Vega's ten place table of logarithms, except in the few cases where a greater number of places was necessary, and then recourse was had to Thoman's tables. The computation has been done twice.

The total number of papers read was greater by two than the number read at last year's summer meeting. The attendance was the same as last year. The Council announced that the regular October meeting of the Society would be replaced by a special meeting to be held at Princeton, on October 17th, in honor of Profs. Felix Klein and J. J. Thomson, who would be in Princeton at that time as delegates to sesquicentennial celebration of Princeton University.

Thomas S. Fiske.
Columbia University.

A CURVE-TRACING TOP.

Some time ago I constructed a top (since called the gyrograph) for directly mapping out the curves corresponding to the precessional and progressive motion of a spinning body. I have since found the instrument of service in teaching this rather troublesome subject, and I will, therefore, venture to give an account of some of its performances.

The instrument is exceedingly simple, and consists merely of a form of stably spinning top, not too heavy, having a socket at the bottom of the stem for the axial insertion of the pencil on which the top is to spin. Particular care must be taken, however, to have the top well balanced and the pencil centered, and I have, therefore, sketched in the annexed figure the form with which I obtained my best results. Here a is a thin disc or web of tin plate carrying a circular ring (b) of $\frac{1}{8}$-inch copper wire; c is a thin conical brace to sustain the brass tube (d), which holds the pencil (e) normally to the web. The whole

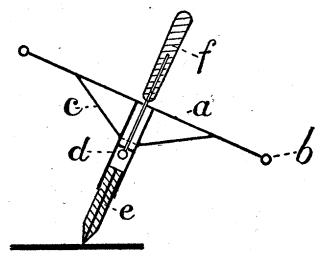

Fig. 1.—Sectional elevation of the top in position for curve tracing (reduced to one-fourth).

is revolvable around the handle (f), the round stem of which passes nicely through central perforations in the web and a diaphragm fixed within the brass tube. The string for spinning is wound around d. The figure is drawn to scale the diameter of the web, being about 6 inches. The weight should not exceed about 5 ounces. It is not desirable to hasten the precessional motion as the curve tracing is best shown with a leisurely swinging top. Furthermore, a tablet at least a foot square, preferably of plate glass, framed and provided with leveling screws, is needed for the accurate delineation of the curves. On this is placed a smooth sheet of white paper. The top, after being spun on the handle, is placed down with its *axis oblique* to the tablet, so that precessional motion may be initiated at the outset.

If the tablet is quite level the curves obtained are spiral with but very slight, if any, lateralness. Two cases are to be distinguished: If the pencil is hard and blunt (preferably cut square off at the lead with a sharp circular edge) precession is markedly accelerated; the top begins with a wide sweep and gradually rising from the oblique to the vertical position, describes a series of spires which converge rapidly from a wide circumference towards a center. If the point is hard and sharp, the top does not rise so